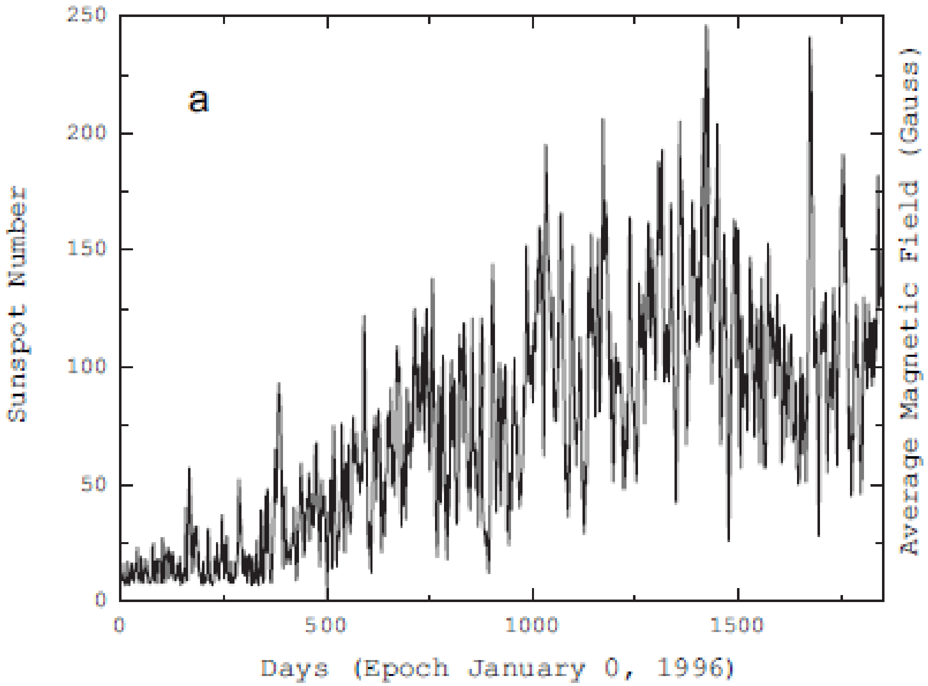
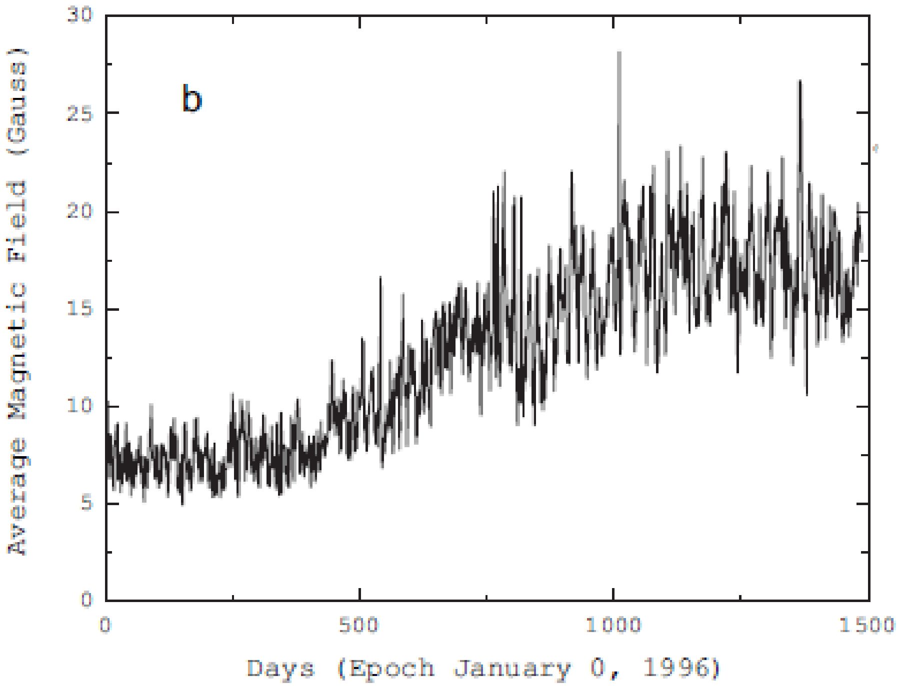
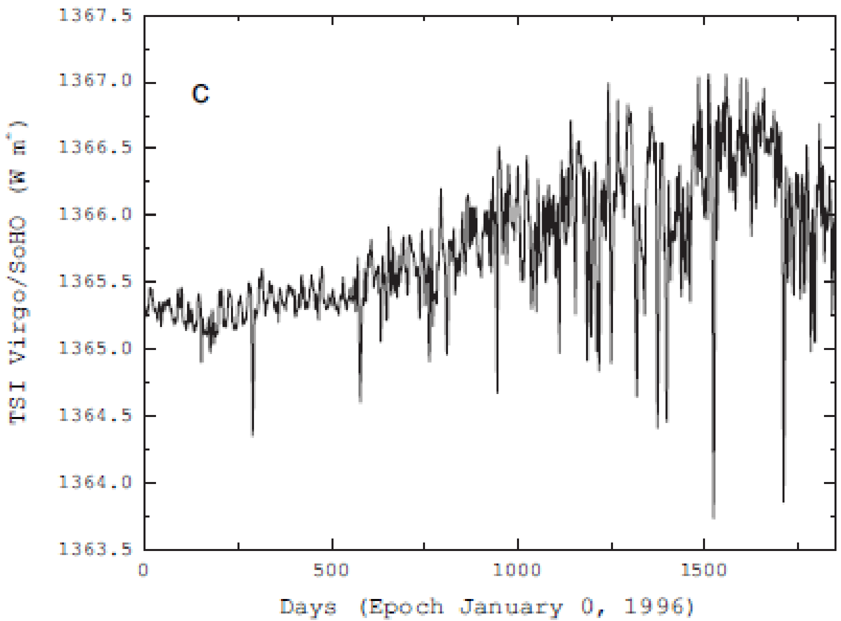


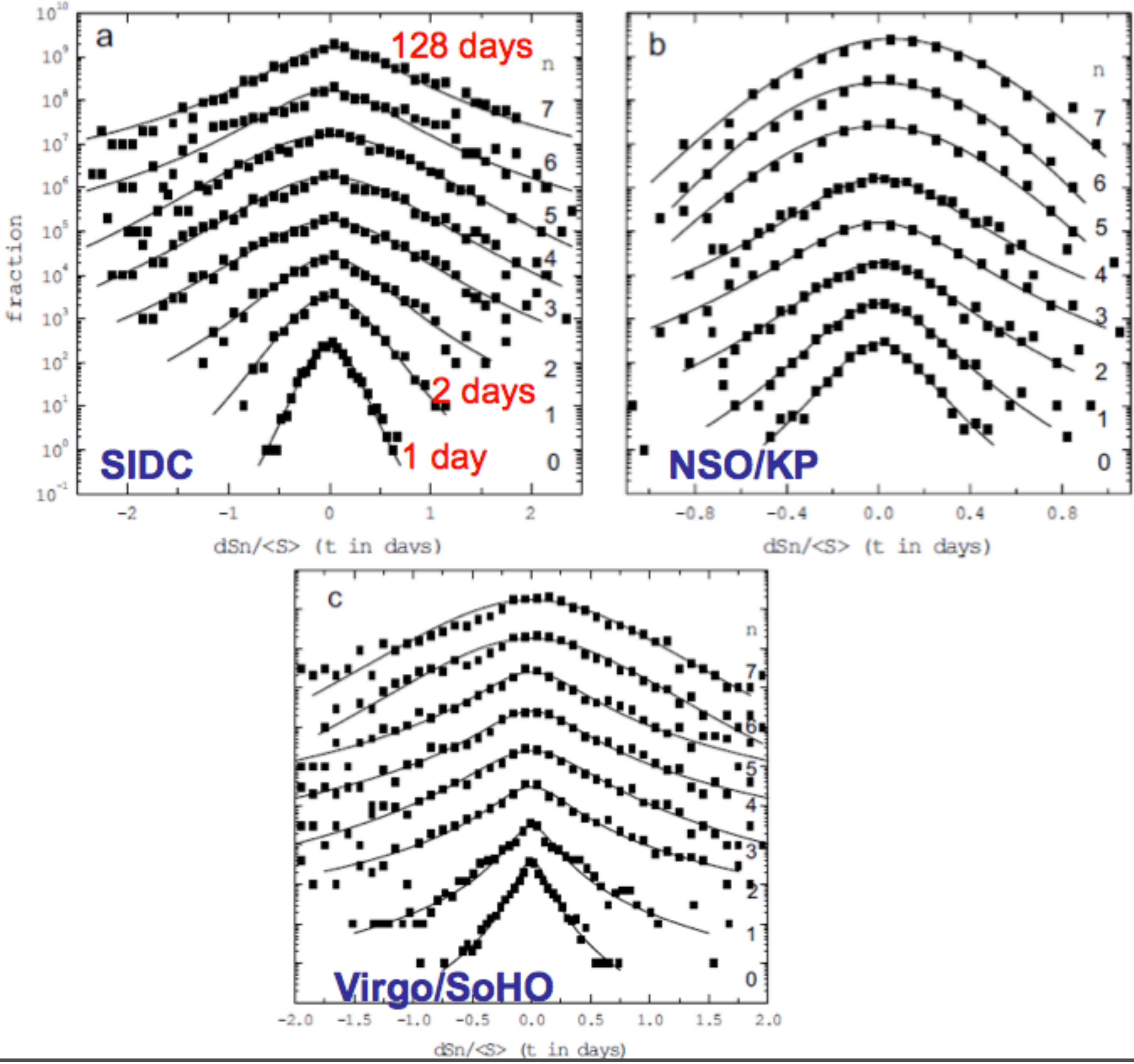
Sunspot number [Sunspot Index Data Center]



Magnetic field
[National Solar Observatory/Kitt Peak]



Total solar irradiance [Virgo/SoHO]



a
128 days
2 days
1 day
SIDC
n
7
6
5
4
3
2
1
0
fraction
dSn/<S> (t in days)
b
NSO/KP
c
Virgo/SoHO

$(\tau = 1 \text{ day})$

	q_{stat}	q_{sen}	q_{rel}
Solar Number [Sunspot Index Data Center]	1.31 ± 0.07	-0.71 ± 0.10	1
Magnetic Field [National Solar Observatory/Kitt Peak]	1.21 ± 0.06	-0.44 ± 0.07	1
Solar Total Irradiance [Virgo/SoHO]	1.54 ± 0.03	-0.52 ± 0.10	1

D.B. de Freitas and J.R. de Medeiros, Europhys Lett 88, 19001 (2010)

PHYSICAL REVIEW D VOLUME 13, NUMBER 2 15 JANUARY 1976

Black holes and thermodynamics*

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(Received 30 June 1975)

A black hole of given mass, angular momentum, and charge can have a large number of different unobservable internal configurations which reflect the possible different initial configurations of the matter which collapsed to produce the hole. The logarithm of this number can be regarded as the entropy of the black hole and is a measure of the amount of information about the initial state which was lost in the formation of the black hole. If one makes the hypothesis that the entropy is finite, one can deduce that the black holes must emit thermal radiation at some nonzero temperature. Conversely, the recently derived quantum-mechanical result that black holes do emit thermal radiation at temperature $\kappa\hbar/2\pi kc$, where κ is the surface gravity, enables one to prove that the entropy is finite and is equal to $c^3A/4G\hbar$, where A is the surface area of the event horizon or boundary of the black hole. Because black holes have negative specific heat, they cannot be in stable thermal equilibrium except when the additional energy available is less than 1/4 the mass of the black hole. This means that the standard statistical-mechanical canonical ensemble cannot be applied when gravitational interactions are important. Black holes behave in a completely random and time-symmetric way and are indistinguishable, for an external observer, from white holes. The irreversibility that appears in the classical limit is merely a statistical effect.

When entropy does not seem extensive

Earlier speculations about the entropy of black holes has prompted an ingenious calculation suggesting that entropy may (in special circumstances) be the same inside and outside an arbitrary boundary.

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. That, of course, is why the entropy of some substance will be quoted as so much per gram, or mole. If you then take two grams, or two moles, of the same material under the same conditions, the entropy will be twice as much. And there should be no confusion about the units; the simple Carnot definition of a change of entropy in a reversible process is the heat transfer divided by the absolute temperature, so that the units of entropy are simply those of energy divided by temperature, joules per degree (kelvin) in the SI system. The definitions of the Gibbs and Helmholtz free energies would be dimensionally discordant for that reason were it not that entropy (S) always turns up multiplied by temperature T. So much will readily be agreed.

Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? By the number of ways in which the constituents of some material (the atoms and molecules) can be rearranged without changing its properties and without energetic consequences. But now there comes a snag.

Like any extensive property, the combined entropy of two separate chunks of material should be the *sum* of the two entropies, but the number of rearrangements of the combined system must be the *product* of the numbers of ways in which the two parts separately can be rearranged. How to reconcile that with extensivity? By supposing entropy is proportional not to the number of rearrangements (technically called 'complexions'), but with the logarithm thereof. And because entropy decreases as disorder increases, the constant of proportionality must be a negative (real) number.

From that it follows that $S = S_0 - K \log N$, where K is a positive constant with the dimensions of entropy, N is a number (without dimensions) measuring disorder and S_0 is an arbitrary constant entropy. All that is simply a precis of the standard introductory chapter in statistical mechanics textbooks, most of which go on to show how to calculate the properties of assemblages of, say, diatomic molecules from a knowledge of their individual behaviour. Because the number of complexions of a particular state of an assemblage is invariably a function of the number (n) of molecules it contains, usually in the form of $n!$, because n is usually large and because $\log(n!)$ can then be approximated by $n \log n$, the extensive property of entropy then follows simply from the appearance of the leading factor n: entropy is proportional to the number of molecules.

That is what the textbooks say. It also makes sense of what is known of the thermodynamics of the real world. In a sample of a diatomic gas, for example, there are vibrations (one) and rotations (two) as well as three rectilinear degrees of freedom. But the problem is to tell how the energy available is distributed among the different degrees of freedom. The arithmetic simplifies marvellously because (in this case) each molecule and each of its degrees of freedom is independent. The best measure of disorder works out at $N = Z^n$, where n is the number of molecules, and where Z, which must be a

A bit of quantum mechanics goes into the argument as well, notably the notion of the density matrix — an artificially constructed operator (on quantum states) that is well suited to the discussion of systems in which one part (say the black hole) is singled out for attention while the remainder (the Universe outside it) is dealt with in less detail, perhaps because some averaging process is appropriate, or because the whole problem may not be calculable at all. (In Dirac's notation, the density matrix corresponding to some state of the whole Universe would be represented as $|1\rangle\langle 1|$, where "1" is simply the name for a particular state of the Universe.) What matters, where entropy is concerned, is that the density matrix, like all matrices, has eigenvalues from which the entropy can be calculated.

So imagine that the Universe is partitioned into two parts by means of a closed boundary of some kind and filled with a

dealt with explicitly, as other entropy calculations are made. And that could be exceedingly important.

John Maddox

Tackled by
- Jacob D. Bekenstein
- Stephen W. Hawking
- Gary W. Gibbons
- Gerard 't Hooft
- Leonard Susskind
- Michael J. Duff
- Juan M. Maldacena
- Thanu Padmanabhan
- Robert M. Wald
- and many others

When entropy does not seem extensive
John Maddox, Nature **365**, 103 (1993)

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. [...] Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? [...] So why is the entropy of a black hole proportional to the square of its radius, and not to the cube of it? To its surface area rather than to its volume?

RAPID COMM

PHYSICAL REVIEW D **73,** 121701(R) (2006)

How robust is the entanglement entropy-area relation?

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(Received 30 November 2005; revised manuscript received 24 May 2006; published 28 June 2006)

We revisit the problem of finding the entanglement entropy of a scalar field on a lattice by tracing over its degrees of freedom inside a sphere. It is known that this entropy satisfies the area law—entropy proportional to the area of the sphere—when the field is assumed to be in its ground state. We show that the area law continues to hold when the scalar field degrees of freedom are in generic coherent states and a class of squeezed states. However, when excited states are considered, the entropy scales as a lower power of the area. This suggests that, for large horizons, the ground state entropy dominates, whereas entropy due to excited states gives power-law corrections. We discuss possible implications of this result to black hole entropy.

The area (as opposed to volume) proportionality of BH entropy has been an intriguing issue for decades.

PHYSICAL REVIEW D **83,** 064034 (2011)

Ideal gas in a strong gravitational field: Area dependence of entropy

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(Received 24 January 2011; published 24 March 2011)

We study the thermodynamic parameters like entropy, energy etc. of a box of gas made up of indistinguishable particles when the box is kept in various static background spacetimes having a horizon. We compute the thermodynamic variables using both statistical mechanics as well as by solving the hydrodynamical equations for the system. When the box is far away from the horizon, the entropy of the gas depends on the volume of the box except for small corrections due to background geometry. As the box is moved closer to the horizon with one (leading) edge of the box at about Planck length (L_p) away from the horizon, the entropy shows an area dependence rather than a volume dependence. More precisely, it depends on a small volume $A_\perp L_p/2$ of the box, up to an order $\mathcal{O}(L_p/K)^2$ where $A_\perp$ is the transverse area of the box and K is the (proper) longitudinal size of the box related to the distance between leading and trailing edge in the vertical direction (i.e. in the direction of the gravitational field). Thus the contribution to the entropy comes from only a fraction $\mathcal{O}(L_p/K)$ of the matter degrees of freedom and the rest are suppressed when the box approaches the horizon. Near the horizon all the thermodynamical quantities behave as though the box of gas has a volume $A_\perp L_p/2$ and is kept in a Minkowski spacetime. These effects are: (i) purely kinematic in their origin and are independent of the spacetime curvature (in the sense that the Rindler approximation of the metric near the horizon can reproduce the results) and (ii) observer dependent. When the equilibrium temperature of the gas is taken to be equal to the horizon temperature, we get the familiar $A_\perp/L_p^2$ dependence in the expression for entropy. All these results hold in a $D+1$ dimensional spherically symmetric spacetime. The analysis based on methods of statistical mechanics and the one

lead to the same result

Thus the extensive property of entropy no longer holds and one can check that it does not hold even in the weak field limit discussed above when $L \gg \lambda$ that is, when gravitational effects subdue the thermal effects along the direction of the gravitational field.

SINCE THE PIONEERING BEKENSTEIN-HAWKING RESULTS, PHYSICALLY MEANINGFUL EVIDENCE HAS ACCUMULATED (e.g., HOLOGRAPHIC PRINCIPLE) WHICH MANDATES THAT

$$\ln W_{black\ hole} \propto \text{AREA}$$

THIS IS PERFECTLY ADMISSIBLE AND MOST PROBABLY CORRECT.

HOWEVER,

IS THIS QUANTITY THE THERMODYNAMICAL ENTROPY???

REVIEWS OF MODERN PHYSICS, VOLUME 82, JANUARY–MARCH 2010

Colloquium: Area laws for the entanglement entropy

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and Institute for Mathematical Sciences, Imperial College London, Exhibition Road, London SW7 2PG, United Kingdom

M. Cramer and M. B. Plenio

Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2BW, United Kingdom
and Institut für Theoretische Physik, Albert-Einstein-Allee 11, Universitat Ulm, D-89069 Ulm, Germany

(Published 4 February 2010)

Physical interactions in quantum many-body systems are typically local: Individual constituents interact mainly with their few nearest neighbors. This locality of interactions is inherited by a decay of correlation functions, but also reflected by scaling laws of a quite profound quantity: the entanglement entropy of ground states. This entropy of the reduced state of a subregion often merely grows like the boundary area of the subregion, and not like its volume, in sharp contrast with an expected extensive behavior. Such "area laws" for the entanglement entropy and related quantities have received considerable attention in recent years.

Eur. Phys. J. C (2013) 73:2487
DOI 10.1140/epjc/s10052-013-2487-6

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Black hole thermodynamical entropy

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[2]Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

Abstract As early as 1902, Gibbs pointed out that systems whose partition function diverges, e.g. gravitation, lie outside the validity of the Boltzmann–Gibbs (BG) theory. Consistently, since the pioneering Bekenstein–Hawking results, physically meaningful evidence (e.g., the holographic principle) has accumulated that the BG entropy S_{BG} of a $(3+1)$ black hole is proportional to its area L^2 (L being a characteristic linear length), and not to its volume L^3. Similarly it exists the *area law*, so named because, for a wide class of strongly quantum-entangled d-dimensional systems, S_{BG} is proportional to $\ln L$ if $d=1$, and to L^{d-1} if $d>1$, instead of being proportional to L^d ($d \geq 1$). These results violate the extensivity of the thermodynamical entropy of a d-dimensional system. This thermodynamical inconsistency disappears if we realize that the thermodynamical entropy of such nonstandard systems is *not* to be identified with the BG *additive* entropy but with appropriately generalized *nonadditive* entropies. Indeed, the celebrated usefulness of the BG entropy is founded on hypothesis such as relatively weak probabilistic correlations (and their connections to ergodicity, which by no means can be assumed as a general rule of nature). Here we introduce a generalized entropy which, for the Schwarzschild black hole and the area law, can solve the thermodynamic puzzle.

ENTROPIES

$$S_{BG} = k_B \sum_{i=1}^{W} p_i \ln \frac{1}{p_i} \qquad \rightarrow \textit{additive}$$

$$S_q = k_B \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i} \qquad (S_1 = S_{BG}) \rightarrow \textit{nonadditive} \text{ if } q \neq 1$$ C. T. (1988)

$$S_\delta = k_B \sum_{i=1}^{W} p_i \left(\ln \frac{1}{p_i} \right)^\delta \qquad (S_1 = S_{BG}) \rightarrow \textit{nonadditive} \text{ if } \delta \neq 1$$ C. T. (2009)

$$S_{q,\delta} = k_B \sum_{i=1}^{W} p_i \left(\ln_q \frac{1}{p_i} \right)^\delta \qquad (S_{q,1} = S_q;\ S_{1,\delta} = S_\delta;\ S_{1,1} = S_{BG})$$ C. T. (2011)

$$\rightarrow \textit{nonadditive} \text{ if } (q,\delta) \neq (1,1)$$

C. T. and L.J.L. Cirto, Eur Phys J C 73, 2487 (2013)

See also: R. Hanel and S. Thurner, EPL **93**, 20006 (2011) and EPL **96**, 50003 (2011)
R. Hanel, S. Thurner and M. Gell-Mann, PNAS **111**, 6905 (2014)

Various arguments (phenomenological, holographic principle, string theory, area law, etc) yield

$$S_{BG}(L) \equiv k_B \ln W(L) \propto L^{d-1} \quad (d>1)$$

hence

$$W(L) \propto \Phi(L)\; \nu^{L^{d-1}} \quad \left(\text{with } \lim_{L\to\infty} \frac{\ln\, \Phi(L)}{L^{d-1}} = 0;\ \text{e.g.}, \Phi(L) \propto L^{\rho} \right)$$

hence, for $d>1$, the entropy which is extensive is S_δ with $\delta = \dfrac{d}{d-1}$

i.e., $$S_{\delta=d/(d-1)}(L) = k_B \sum_{i=1}^{W(L)} p_i \left(\ln \frac{1}{p_i} \right)^{\frac{d}{d-1}} \propto L^d \quad (d>1)$$

Consequently $$S_{\delta=3/2}^{black\ hole}(L) = k_B \sum_{i=1}^{W(N)} p_i \left(\ln \frac{1}{p_i} \right)^{\frac{3}{2}} \propto L^3 \quad !!!$$

C.T. and L.J.L. Cirto, Eur. Phys. J. C **73**, 2487 (2013)

PHYSICAL REVIEW D **88,** 083534 (2013)

Entropic cosmology for a generalized black-hole entropy

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(Received 23 July 2013; published 28 October 2013)

An entropic-force scenario, i.e., entropic cosmology, assumes that the horizon of the Universe has an entropy and a temperature. In the present paper, in order to examine entropic cosmology, we derive entropic-force terms not only from the Bekenstein entropy but also from a generalized black-hole entropy proposed by Tsallis and Cirto [Eur. Phys. J. C **73**, 2487 (2013)]. Unlike the Bekenstein entropy, which is proportional to area, the generalized entropy is proportional to volume because of appropriate nonadditive generalizations. The entropic-force term derived from the generalized entropy is found to behave as if it were an extra driving term for bulk viscous cosmology, in which a bulk viscosity of cosmological fluids is assumed. Using an effective description similar to bulk viscous cosmology, we formulate the modified Friedmann, acceleration, and continuity equations for entropic cosmology. Based on this formulation, we propose two entropic-force models derived from the Bekenstein and generalized entropies. In order to examine the properties of the two models, we consider a homogeneous, isotropic, and spatially flat universe, focusing on a single-fluid-dominated universe. The two entropic-force models agree well with the observed supernova data. Interestingly, the entropic-force model derived from the generalized entropy predicts a decelerating and accelerating universe, as for a fine-tuned standard ΛCDM (lambda cold dark matter) model, whereas the entropic-force model derived from the Bekenstein entropy predicts a uniformly accelerating universe.

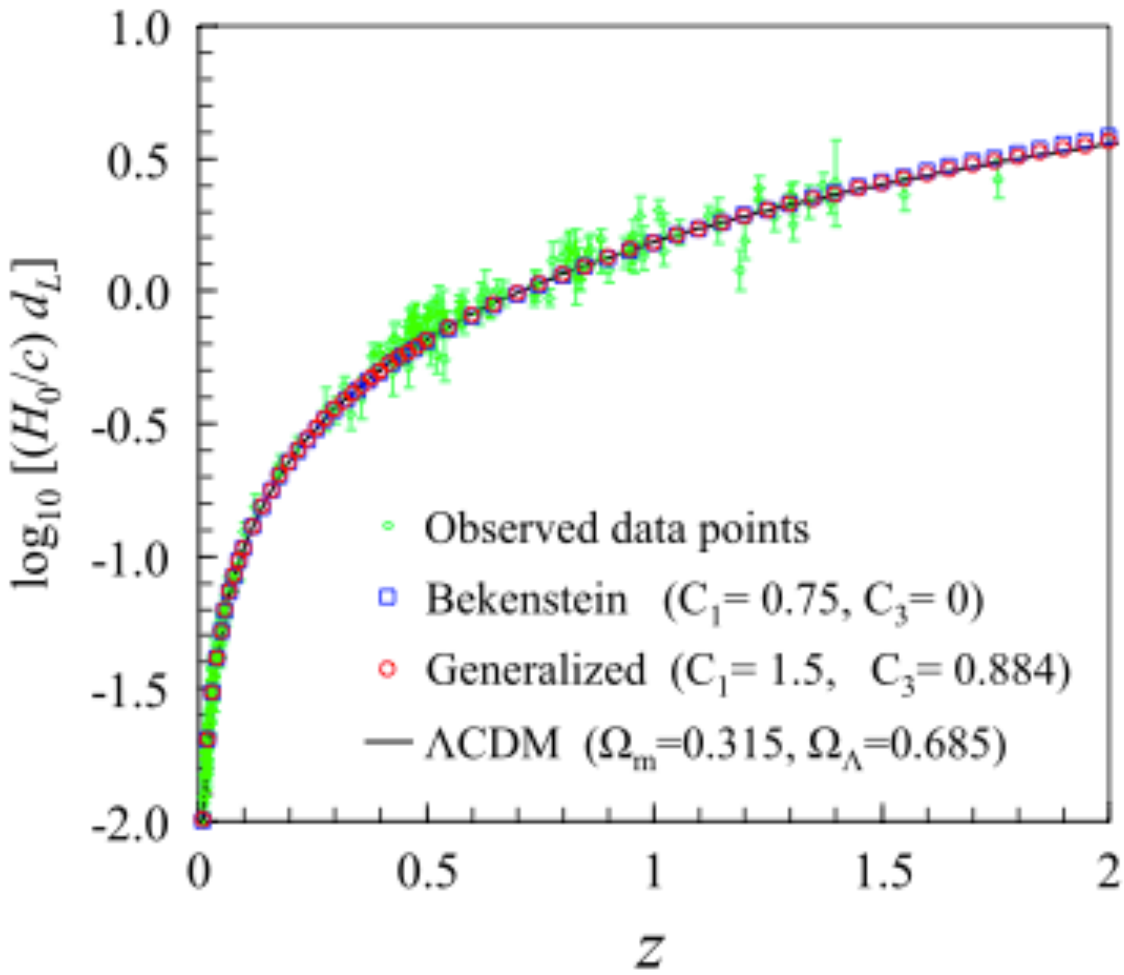

FIG. 1 (color online). Dependence of luminosity distance d_L on redshift z. Here, Bekenstein and Generalized indicate the information for the entropic-force models derived from the Bekenstein and generalized entropies, respectively. The open diamonds with error bars are supernova data points taken from Ref. [3]. For supernova data points, H_0 is set to be 67.3 km/s/Mpc based on the Planck 2013 results [6].

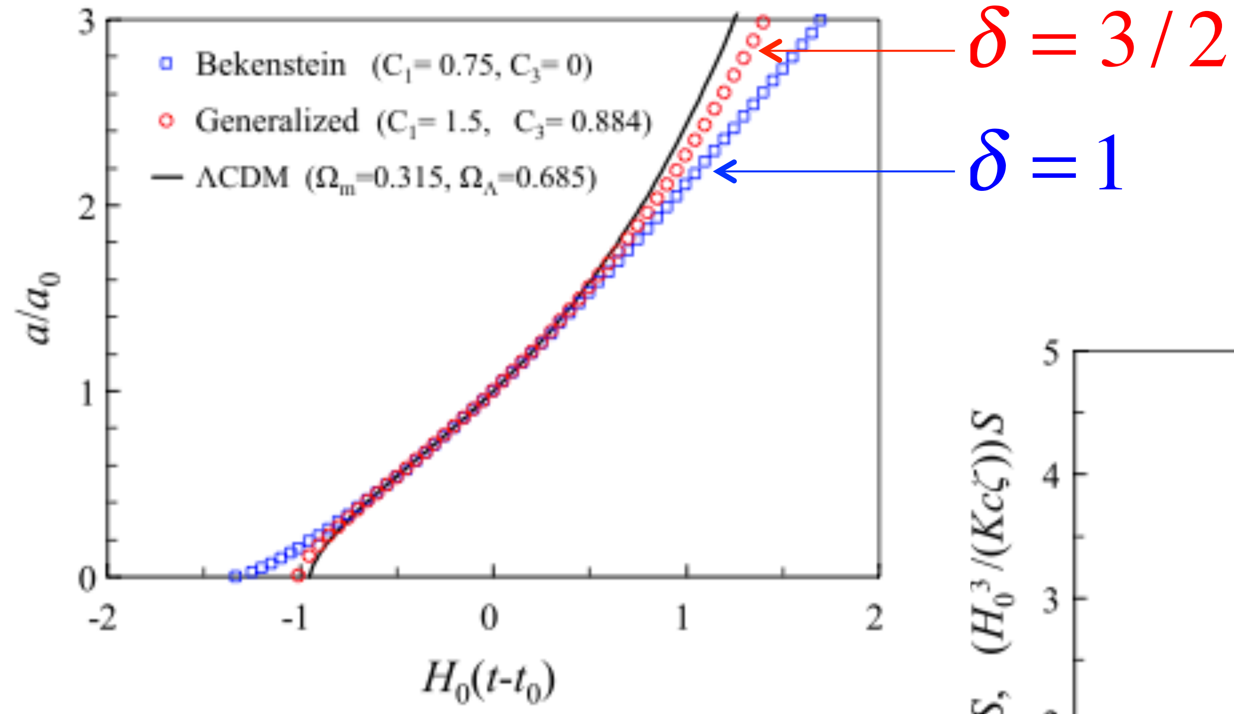

FIG. 2 (color online). Time evolution of normalized scale factor a/a_0. The horizontal axis is normalized as $H_0(t-t_0)$. Here, Bekenstein and Generalized indicate the information for the entropic-force models derived from the Bekenstein and generalized entropies, respectively.

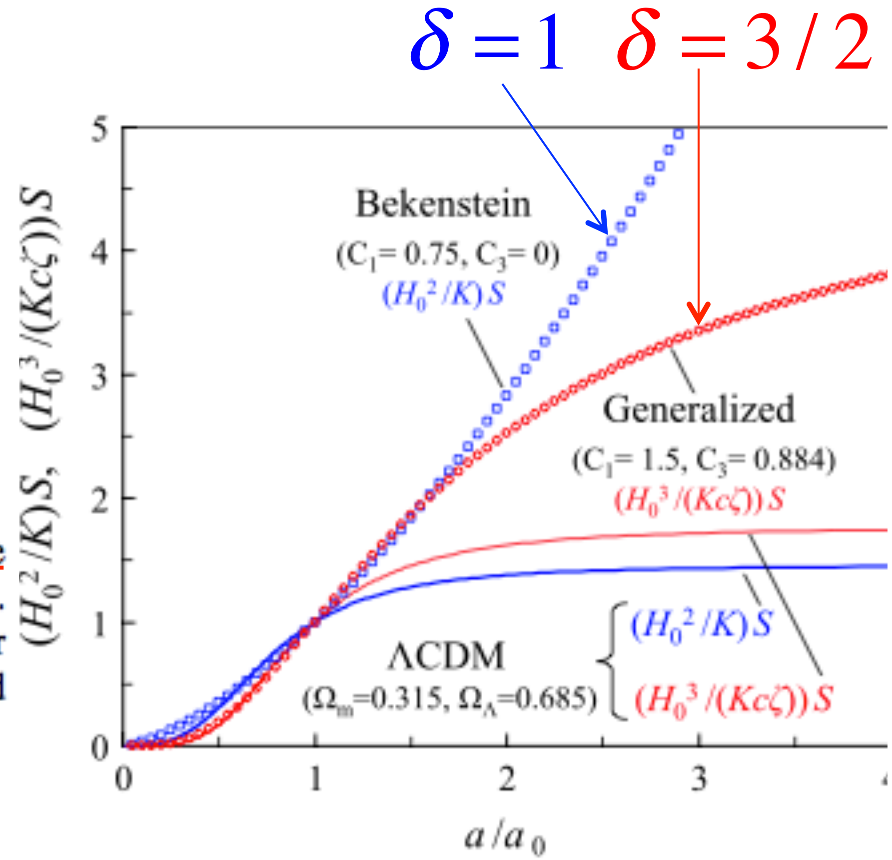

FIG. 3 (color online). Evolutions of the Bekenstein and eralized entropies. The vertical axis represents $(H_0^2/K)S$ $(H_0^3/(Kc\zeta))S$, for the Bekenstein and generalized entropic- models, respectively. The solid lines represent $(H_0^2/K)S$ $(H_0^3/(Kc\zeta))S$ for the fine-tuned standard ΛCDM model are numerically calculated from $(H/H_0)^{-2}$ and $(H/H_0)^{-3}$.

PHYSICAL REVIEW E **84**, 021121 (2011)

Group entropies, correlation laws, and zeta functions

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(Received 15 February 2011; revised manuscript received 3 May 2011; published 10 August 2011)

The notion of group entropy is proposed. It enables the unification and generaliztion of many different definitions of entropy known in the literature, such as those of Boltzmann-Gibbs, Tsallis, Abe, and Kaniadakis. Other entropic functionals are introduced, related to nontrivial correlation laws characterizing universality classes of systems out of equilibrium when the dynamics is weakly chaotic. The associated thermostatistics are discussed. The mathematical structure underlying our construction is that of formal group theory, which provides the general structure of the correlations among particles and dictates the associated entropic functionals. As an example of application, the role of group entropies in information theory is illustrated and generalizations of the Kullback-Leibler divergence are proposed. A new connection between statistical mechanics and zeta functions is established. In particular, Tsallis entropy is related to the classical Riemann zeta function.

$$S_q \leftrightarrow \frac{1}{(1-q)^{s-1}}\varsigma(s) \quad (q<1)$$

$$\text{with } \varsigma(s) \equiv \sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_{p\ \text{prime}}\frac{1}{1-p^{-s}}$$

$$= \frac{1}{1-2^{-s}}\frac{1}{1-3^{-s}}\frac{1}{1-5^{-s}}\frac{1}{1-7^{-s}}\frac{1}{1-11^{-s}}\cdots$$

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

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doi: 10.1209/0295-5075/102/28006

www.epljournal.org

On the non-extensivity in Mars geological faults

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published online 3 May 2013

PACS 89.75.Da – Systems obeying scaling laws
PACS 89.75.-k – Complex systems
PACS 96.30.Gc – Mars

Abstract – A non-extensive statistical physics approach is tested for the first time in a planetary scale, for the fault length distribution in Mars estimated a non-extensive q-parameter equal to 1.277 for normal faults and 1.114 for thrust ones. The latter support the conclusion that the fault systems in Mars are subadditive ones in agreement with recent observations for faults in Earth and Valles Marineris extensional province, Mars. In addition, an analysis of the global Mars fault system as a mixed one, consisted of the normal and thrust subsystems with different q-parameters is presented, leading to $q = 1.22$.

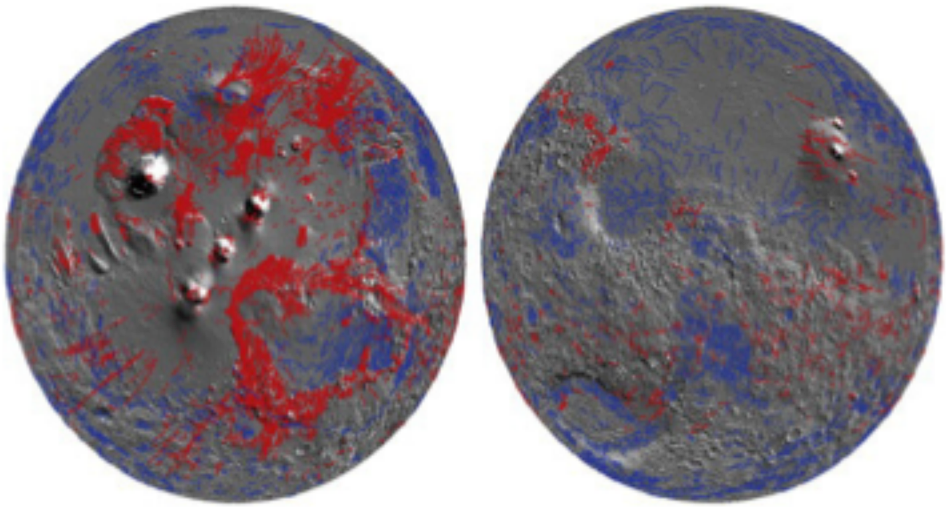
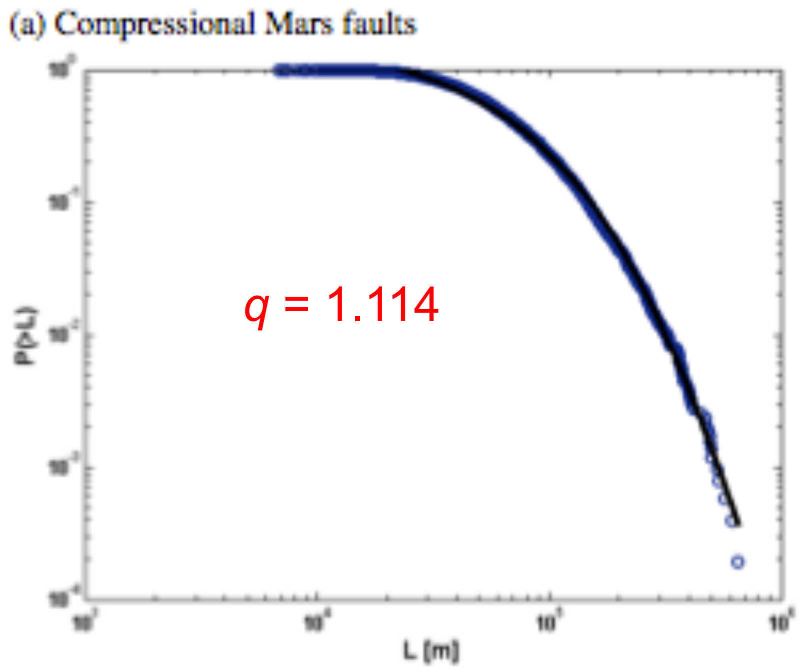
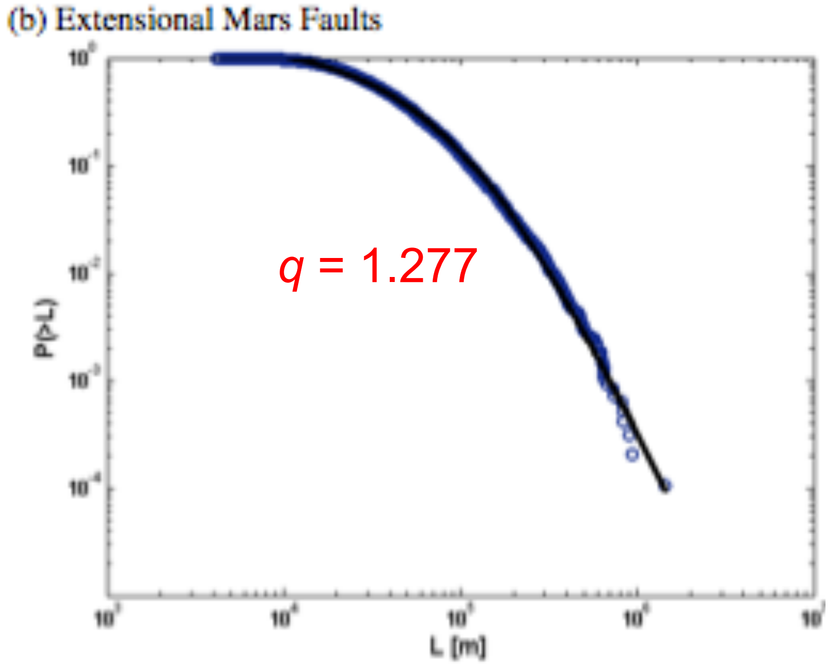


Fig. 1: (Colour on-line) Global distribution of faults on Mars Western hemisphere (left) and eastern hemisphere (right), extracted from [22] and [42]. The extensional faults (in red) are mainly concentrated in the Western hemisphere, while the contractional faults, are located in both Mars hemispheres.





Latin American Journal of Solids and Structures
www.lajss.org

Acoustic Emission Analysis of Cement Mortar Specimens During Three Point Bending Tests

Abstract

This work discusses the experimental results of Acoustic Emission (AE) recordings during repetitive loading/unloading loops of cement mortar beams subjected to three point bending. Six repetitive loading cycles were conducted at a gradually higher load level until the failure of the specimens. The experimental results clearly show the existence and dominance of the Kaiser effect during each loading loop. Regarding the AE data, alternative analysis was conducted using the improved b-value, and the cumulative energy behaviour. Both quantities considered, show qualitative and quantitative characteristics that could be used as pre-failure indicators. In addition, a novel statistical physics analysis involving the AE interevent times was conducted by calculating the cumulative probability function $P(>\delta\tau)$ that follows a q-exponential equation. The entropic index q and the relaxation parameter β_q of this equation show systematic changes during the various stages of the failure process. The last cycle led to a q value equal to 1.42, implying the upcoming fracture which is in good agreement with previous results obtained from a wide range of fractured materials.

Keywords

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Photo 1: Typical white cement specimen.

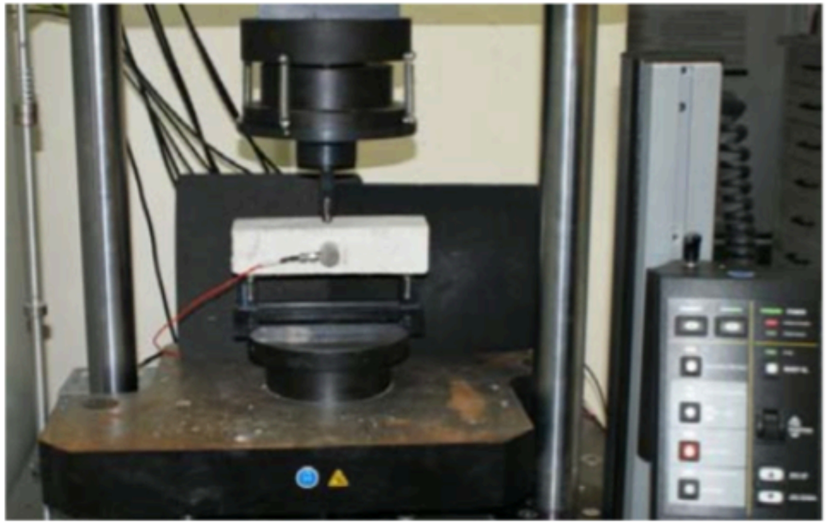
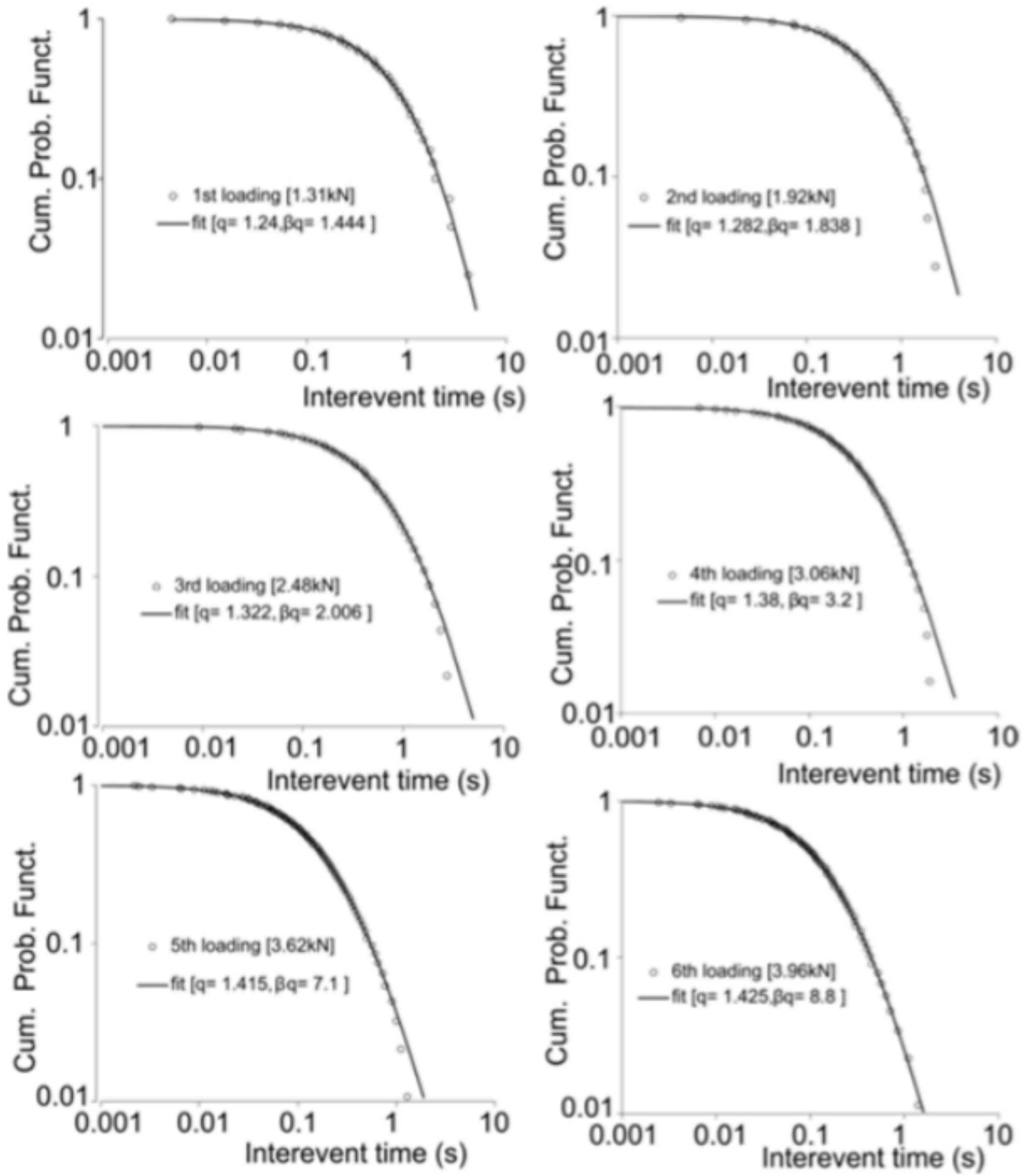


Photo 2: The experimental setup.



1st loading [1.31kN]
fit [q= 1.24,βq= 1.444]
2nd loading [1.92kN]
fit [q= 1.282,βq= 1.838]
3rd loading [2.48kN]
fit [q= 1.322, βq= 2.006]
4th loading [3.06kN]
fit [q= 1.38, βq= 3.2]
5th loading [3.62kN]
fit [q= 1.415,βq= 7.1]
6th loading [3.96kN]
fit [q= 1.425,βq= 8.8]
Cum. Prob. Funct.
Interevent time (s)
1
0.1
0.01
0.001
0.01
0.1
1
10

Eur. Phys. J. Special Topics **226**, 1433–1443 (2017)

DOI: 10.1140/epjst/e2016-60252-2

THE EUROPEAN
PHYSICAL JOURNAL
SPECIAL TOPICS

Review

On the foundations of statistical mechanics*

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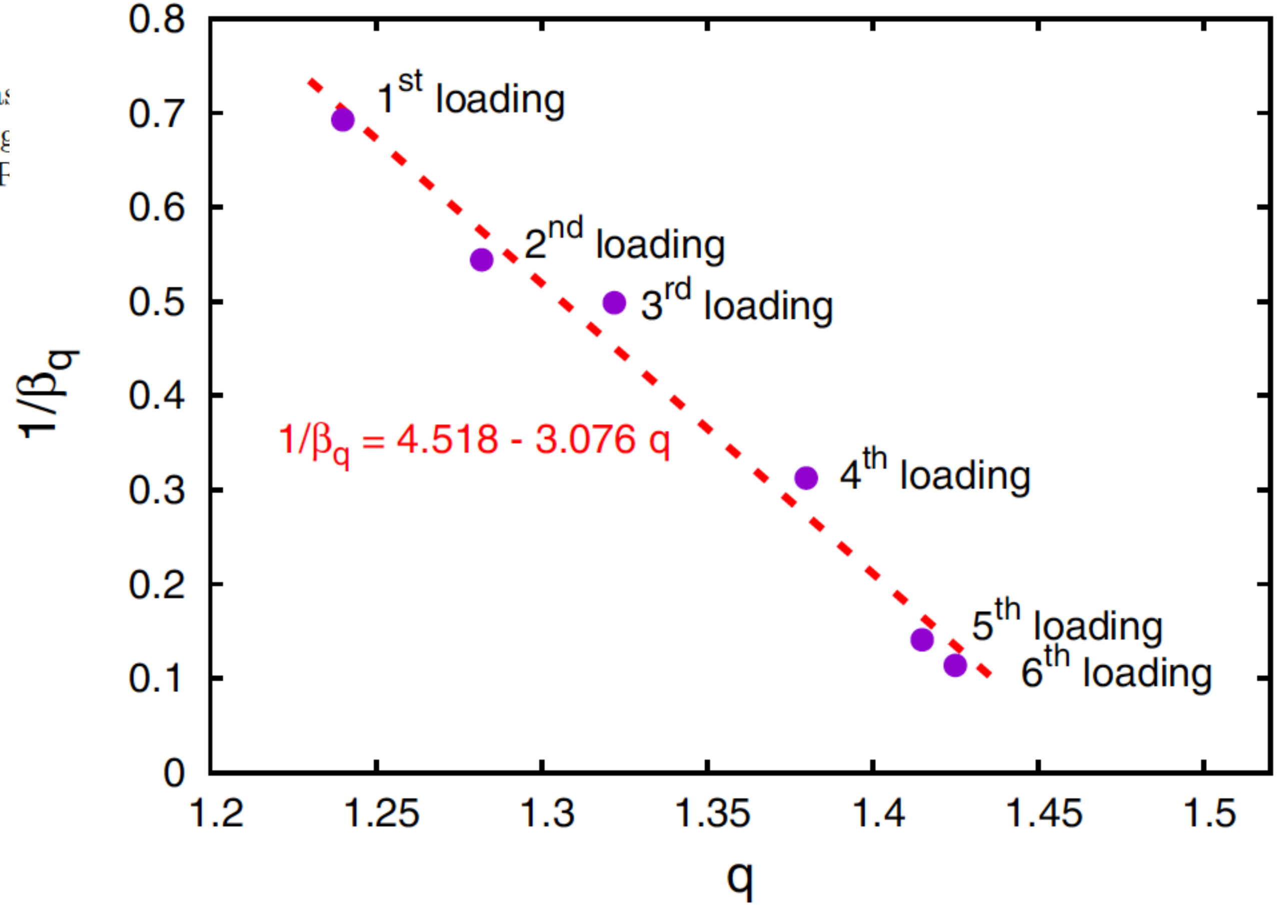
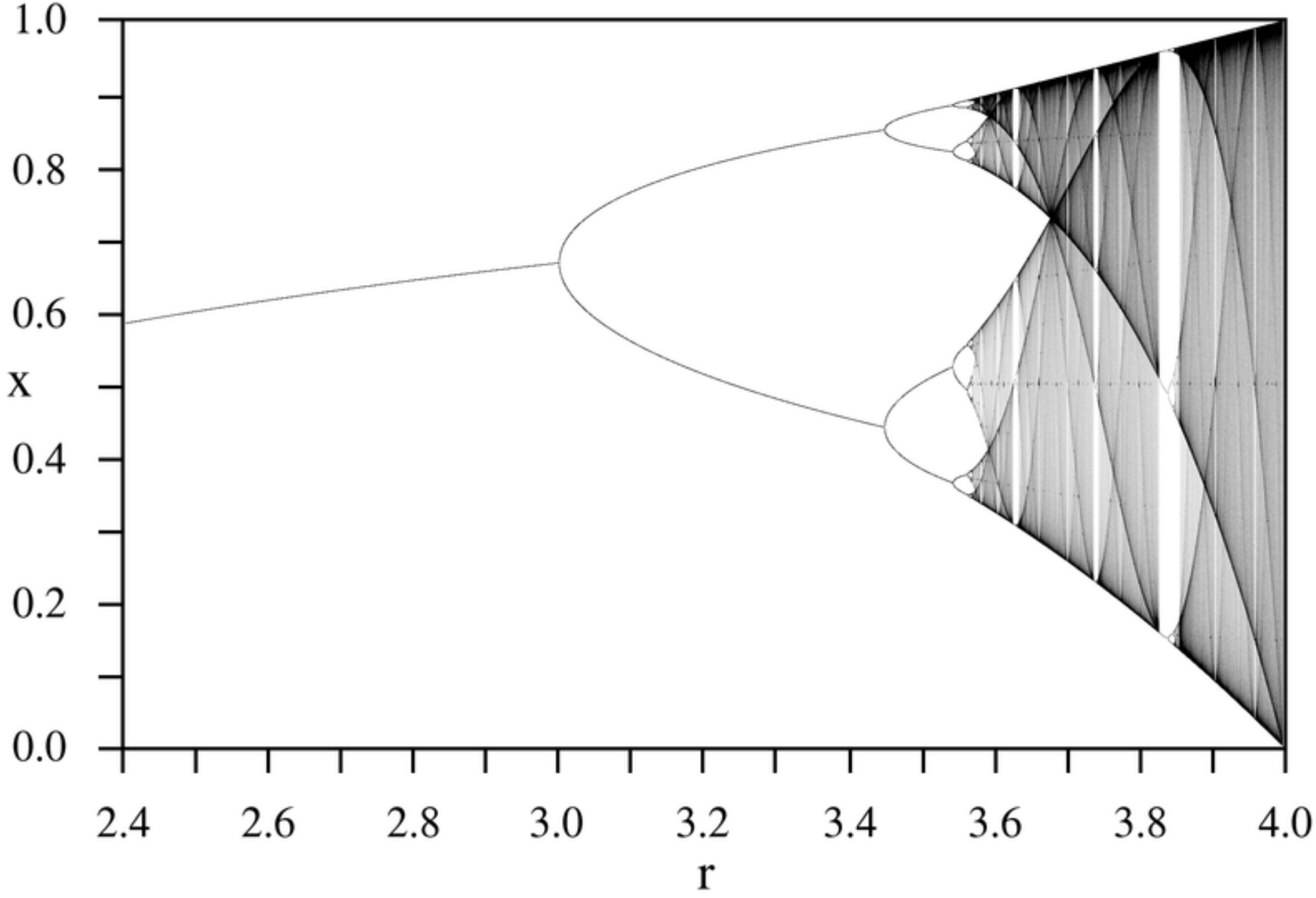
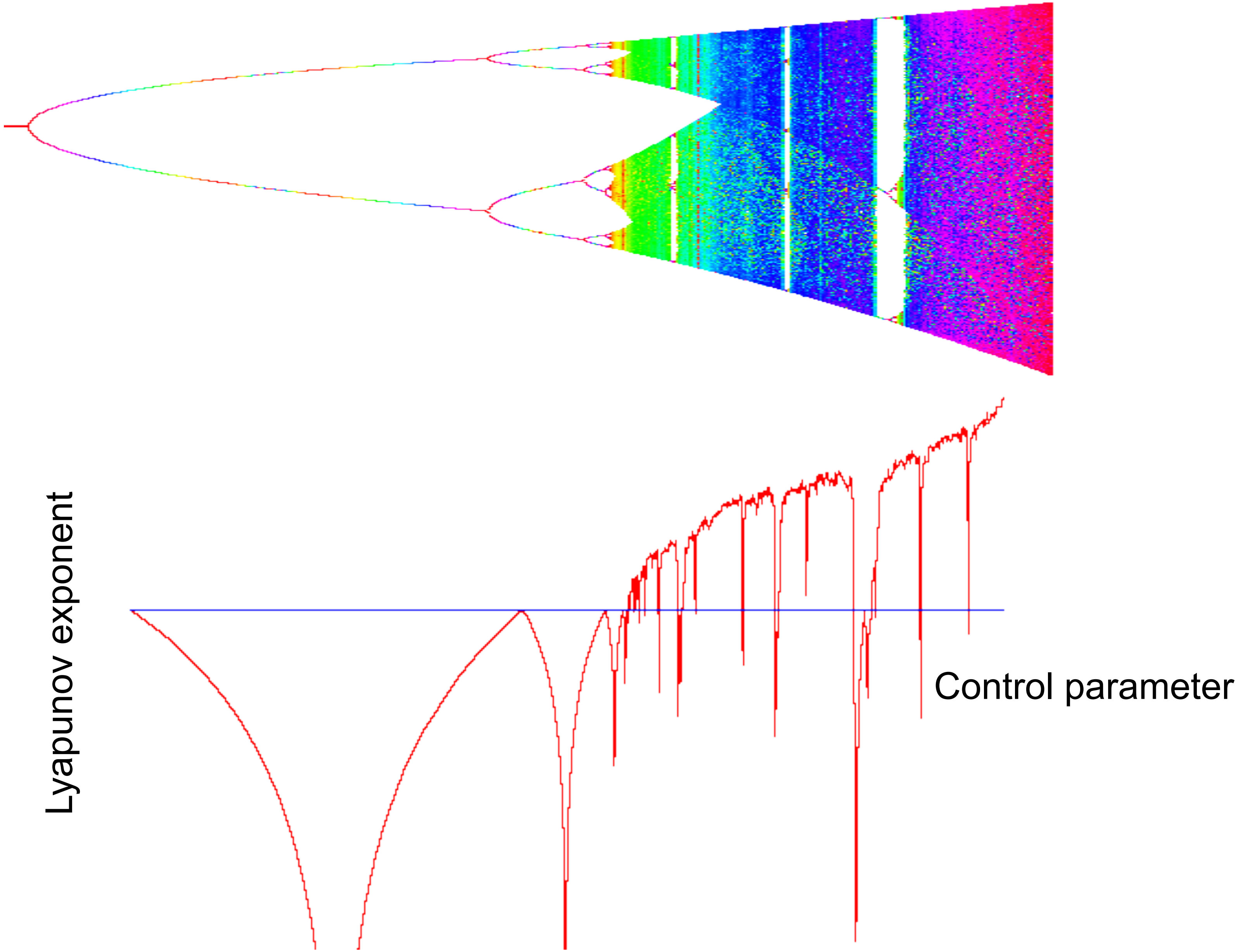


One-dimensional dissipative maps

LOGISTIC MAP:




Lyapunov exponent
Control parameter